CHEM2504 HW 7

Due: Apr 30, 5:00 pm, 2024

The case of an infinite square well whose right wall expands at a *constant* velocity (v) can be solved exactly. A complete set of solutions is

$$\Phi_n(x,t) \equiv \sqrt{\frac{2}{\omega}} \sin\left(\frac{n\pi}{\omega}x\right) e^{i(mvx^2 - 2E_n^i at)/2\hbar\omega} \tag{1}$$

where $\omega(t) \equiv a + vt$ is the width of the well and $E_n^i \equiv n^2\pi^2\hbar^2/2ma^2$ is the nth allowed energy of the *original* well (width a). The *general* solution is a linear combination of the Φ's:

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \Phi_n(x,t) \tag{2}$$

the coefficients c_n are *independent* of t.

1. Check that Equation(1) satisfies the time-dependent Schrödinger equation, with the appropriate boundary conditions.

2. Suppose a particle starts out (t = 0) in the ground state of the initial well:

   $$\Psi(x,0) = \sqrt{\tfrac{2}{a}} \sin\left(\tfrac{\pi}{a}x\right)$$

   Show that the expansion coefficients can be written in the form

   $$c_n = \frac{2}{\pi} \int_0^{\pi} e^{-i\alpha z^2} \sin(nz)\sin(z)\,dz \tag{3}$$

   where $\alpha \equiv mva/2\pi^2\hbar$ is a dimensionless measure of the speed with which the well expands. (Unfortunately, this integral cannot be evaluated in terms of elementary functions.)

3. Suppose we allow the well to expand to twice its original width, so the 'external' time is given by $\omega(T_e) = 2a$. The 'internal' time is the *period* of the time-dependent exponential factor in the (initial) ground state. Determine T_e and T_i, and show that the adiabatic regime corresponds to $\alpha \ll 1$, so that $e^{-i\alpha z^2} \cong 1$ over the domain of integration. Use this to determine the expansion coefficients c_n. Construct $\Psi(x, t)$, and confirm that it is consistent with the adiabatic theorem.

4. Show that the phase factor in $\Psi(x, t)$ can be written in the form

$$\Theta(t) = -\frac{1}{\hbar} \int_0^t E_1(t')dt' \tag{4}$$

where $E_n(t) \equiv n^2\pi^2\hbar^2/2m\omega^2$ is the n^{th} *instantaneous* eigenvalue, at time t. Comment on this result.